**RING THEORY**

**Definition:**

A ring consists of a set R on which addition and multiplication satisfy the following axioms:

* Commutativity:

The elements are commutative under addition that is, x + y = y + x for all elements x and y of R.

* Associativity:

The elements are associative under both addition and multiplication that is,

(x + y) + z = x+(y + z) for all elements x, y and z of R.

x(yz) = (xy)z for all elements x, y and z of R.

* Identity:

There exist an element 0 of R (zero element) with the property that x+0 = x for all elements x of R.

* Inverse:

For a given element x of R, there exists an element –x of R with property x + (-x) = 0 for all elements x of R.

* Distributivity:

x(y + z) = xy + xz and (x + y)z = xz + yz for all elements x, y and z of R.

So from the above axioms we can say that R is an Abelian group under addition and a semi group under multiplication and holds distributive laws.

There is a special condition that if R holds commutativity for multiplication that is xy = yx for all elements x and y of R than it is called a Commutative Ring.

**Examples:**

A set of integers is a commutative ring.

A set of rational numbers, real numbers and complex numbers all are examples of commutative ring.

The set of nxn matrices over the field of real numbers is a non-commutative ring.

**Terms:**

* **Unit Element of a Ring:**

A non-zero element of R is called Unit if it has a multiplicative inverse in R that is for every x element (x is not zero) there exist (1/x) such that x(1/x)=1.

* **Division Ring or Skew Field:**

A ring R is called a division ring if all the non-zero elements of R has its multiplicative inverse in R that is each non-zero element of R is unit.

* **Field:**

A commutative division ring is called a field.

* **Zero Divisor:**

If R is a commutative ring then a non-zero element a is called a zero divisor if there is non-zero element b such that ab = 0

**Ideals of Rings:**

A subset of the ring is said to be an ideal if the following conditions are satisfied:

1. There must be the zero-element in the subset.
2. For every x and y elements of subset x + y must also belong to the subset.
3. Additive Inverse must exist that is for each x there must be a –x.
4. For every x and y elements of subset x.y and y.x must also belong to the subset.